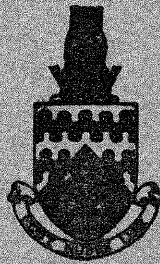




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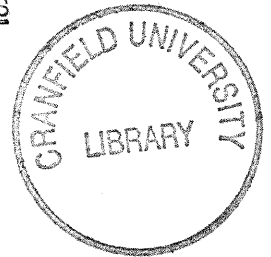


USE OF FORMULA FOR CALCULATING
ECONOMIC BATCH SIZES

by

A. ROBINSON

STATISTICS AND OPERATIONAL
RESEARCH SECTION

THE COLLEGE OF AERONAUTICSGRANFIELDInterim Report on the Research into the use of
Formula for Calculating Economic Batch Sizes

- by -

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STATISTICS AND OPERATIONAL
RESEARCH SECTIONSUMMARY

This report analyses the replies and subsequent conversations held with companies. Based on this analysis it is shown that various factors have to be considered when deciding what is an Economic Batch size, and that the batch size given by the formula $Q = \sqrt{\frac{200AR}{U_t I}}$ is not necessarily the size which provides the best economic gain to a company.

A relationship is formulated between the percentage increase in total unit cost for given variations from the batch size: $\sqrt{\frac{200AR}{U_t I}}$ so that companies may determine what alteration to this batch size is acceptable or desirable under given operating conditions.

The factors to be included in the terms A (set up cost) U_t (unit manufacturing cost), and I (holding charges expressed as a percentage of total manufacturing costs) are discussed, and practical methods of calculating Economic Batch Quantities discussed.

Finally a review is made of further research work required.

LIST OF SYMBOLS

A	Total cost of setting-up including overheads or placing a purchase order
B	Ratio of overhead costs in schedule B to total overheads as per Appendix 3B
C	Ratio of overhead costs in schedule C to total overheads as per Appendix 3C
D	Ratio of overhead costs in schedule D to total overheads as per Appendix 3D
H	Total overheads
I	Total cost of holding stock expressed as a percentage of the total manufacturing cost of the component
I_s	Cost of storing stock expressed as a percentage of the total manufacturing cost of the component
K	The quantity $2 + \frac{QU_t}{A} + \frac{A}{QU_t}$ (see Appendix 2)
L	Direct Labour cost per hour
M	Component material cost
m	ratio $\frac{\text{unit manufacturing time}}{\text{total operating time}}$
N	Number of orders issued per machine/operator per budget period
n	Number of orders issued per machine/operator per hour
P	Percentage increase in Total Unit Cost over minimum cost for quantities above or below the Minimum Cost Quantity
Q	Minimum cost quantity
R	Annual rate of usage
S	Storage cost
T_s	Set up Time
T_t	Total number of operating hours in the overhead budget period
T_u	Time for unit manufacture
U	Unit direct labour cost including overheads

List of Symbols (Continued)

U_t Total of direct labour including overheads and unit material cost

x ratio $\frac{\text{batch quantity}}{\text{minimum cost quantity}}$

y ratio $\frac{\text{overhead cost}}{\text{direct labour cost}}$

PART I. INTRODUCTION AND ACKNOWLEDGEMENTS

1. In March 1959 a request was sent to 50 firms in a wide variety of industries, for information on their methods of calculating Batch Sizes. The introductory letter, data and questionnaire are reprinted in Appendix 1.

So far some 16 firms have shown interest or co-operated by sending in completed questionnaires and/or providing useful comments on the whole problem of Economic Batch Sizes. The range of industries covered by these firms include manufacturers of Power generating plant, Mechanical handling equipment, Aircraft, Metals, Tubes, Furniture and Pharmaceuticals, together with general engineering companies. The Department of Economics and Production thank those concerned for their assistance.

PART II. ANALYSIS OF INFORMATION RECEIVED

2. THE PRESENT USE OF FORMULAE AND SIMILAR METHODS TO OBTAIN ECONOMIC BATCH SIZES

It would appear that of the companies who replied to the questionnaire, some half dozen use formulae or previously established tables or nomograms for obtaining Economic Batch Sizes for controlling at least part of their inventories.

The main reasons advanced by some of the other companies for not using any specific method were :

- (a) The effect of large batch sizes with long manufacturing time cycles delaying the production of other items, thereby causing increases in work-in-progress and bad maintenance of delivery promises.
- (b) Economic Batch Sizes of components solely used in one product may not be multiples or sub-multiples of that final product quantity, hence odd quantities of components will remain in stock.
- (c) The size of batches can be too large requiring finance in excess of the available capital.
- (d) The stock life of the product would be too long for flexibility in meeting changing market conditions.
- (e) The life of the tool may not last the manufacturing run, and break down of the set up would be required for tool sharpening. Thus the quantity affects the set up costs causing difficulty in calculation.

- (f) The size of the batches can be too small, requiring too many set ups. This high total set up time for a given programmed quantity, together with the manufacturing time may exceed the total available plant capacity. This same principle applies to all other production facilities, e.g. supervision, production control, inspection, etc.
- (g) Difficulty, or clerical effort involved, in assessing the values of A.U.I. and R.
- (h) The shop floor area is insufficient to house Economic Batch Sizes during manufacture.
- (i) Difficulty by the concerned staff in applying formulae.

These reasons have been discussed in subsequent sections of this report. Part III deals with paragraphs a to f; Part IV deals with paragraphs g and h and Part V is concerned with paragraph i.

3. OTHER CONSIDERATIONS DECIDING WHAT IS AN ECONOMIC BATCH SIZE

It was also advanced that the Minimum Cost Quantity, given by the formula $\sqrt{\frac{200AR}{U_t I}}$, is not necessarily the most economical quantity, as other considerations enter into stock situations, e.g. increasing the return on capital invested in stock, minimising stock, maximising machine utilisation, or meeting delivery promises. These considerations are dealt with further in Part III. To avoid any pre-judging of a suitable formula for the Economic Batch Size, the quantity expressed by the formula $\sqrt{\frac{200AR}{U_t I}}$ will be referred

throughout this report as the Minimum Cost Quantity, and the term Economic Batch Quantity is that quantity which gives the concerned company the maximum overall financial and economic gain.

4. COMPARISON OF ACTUAL BATCH SIZES ORDERED COMPARED WITH CALCULATED MINIMUM COST QUANTITIES

From an analysis of the replies to the questionnaire, it appears that for high cost items the ordered quantity was smaller than the Minimum Cost Quantity.

For intermediate cost value some companies ordered batch sizes which did not vary more than 20% above or below the calculated Minimum Cost Quantities; whereas other companies ordered quantities which were, in general, smaller than Minimum Cost Quantities, often as little as $1/3$ to $1/5$ the Minimum Cost Quantity.

For low cost items there was no consistent trend, some quantities ordered were 3 or 4 times the calculated Minimum Cost Quantity, other quantities ordered were half the Minimum Cost Quantity. However, very considerable variations from Minimum Cost Quantities occurred when the size of batches of components were related to the quantity of final products, even when normally good correlation between ordered quantities and Minimum Cost Quantities existed in a company. For example if the quantity of the final product was 100, components would be ordered in 100's or multiples thereof, even if the Minimum Cost Quantity was as low as 16.

Some companies applied an upper limit to the normal life of a batch e.g. 1 year's supply and one company also fixed a lower limit to the life of a batch, e.g. 6 weeks supply.

5. DIFFICULTIES IN ASSESSING VALUES OF U_t . A. I. R.

From the replies received companies appear to experience little difficulty in general in assessing the values of U_t (Unit Cost) and A (Set up Cost), though it was often pointed out that assessing and entering this information on the appropriate cards would involve considerable clerical effort which would have to be justified by associated savings. There appears to be conflicting opinions as to whether U (the unit labour cost) and A (the Set up cost) should have overheads added to them. In Part IV of this report an analysis of overheads has been made, and reasons advanced for the inclusion of normal overhead rates in the values of U_t and A.

However, it appears that the assessment of I (the percentage rate on the value of stocks for stock holding charges) causes companies considerable difficulties. The value of I in the various firms ranged from 8 to 16%, each company having one value, except in one case, where the values of I varied in intervals of 2%. A qualitative analysis of the method of assessing I has been included in Part IV.

Certain companies, particularly those engaged in job and small batch production, stated that it was difficult to assess the value of R (the annual rate of usage). Also it was stated that, even when an average annual rate was known, considerable seasonal variations occurred which would make a universal all-the-year-round application of a formula for Economic Batch Size inaccurate and impracticable.

PART III. FACTORS AFFECTING THE DETERMINATION OF AN ECONOMIC BATCH SIZE

6. INTRODUCTION

It has been suggested that the size of a batch which is most economic from an overall company view is not necessarily the quantity with the minimum total unit cost including set up and holding charges. Other factors such as :-

The rate of return on capital employed

Flexibility of the composition of a company's stock to meet changing market conditions

Keeping stocks of components forming assemblies in balance with one another

sometimes makes the quantity with the best economic gain a different quantity than the Minimum Cost Quantity.

Further factors may limit the size of the batch to quantities smaller than the Minimum Cost Quantity, e.g.

Insufficient Capital to finance the Minimum Cost Quantity.

The Minimum Cost Quantity may be so large as to employ the available capacity of a machine group for several weeks causing delays to other products, and thereby increasing delivery dates and increasing work-in-progress.

This part of the report considers these and similar factors and shows that, except in a few cases, the most suitable economic batch size is less than the Minimum Cost Quantity.

The effect on total unit cost of increasing or decreasing the Minimum Cost Quantity has been investigated, and a formula has been derived as set out in Appendix 2.

7. FACTORS MAKING THE ECONOMIC BATCH SIZE DIFFERENT FROM THE MINIMUM COST QUANTITY

The factors which make the Economic Batch Quantity different from the Minimum Cost Quantity can be classified into three groups :-

- (a) Factors which make the Economic Batch Size above the Minimum Cost Quantity.
- (b) Factors which make the Economic Batch Size below the Minimum Cost Quantity.

- (c) Factors which make the Economic Batch Size either above or below the Minimum Cost Quantity according to circumstances.

(a) Factors which make the Economic Batch Size above the Minimum Cost Quantity

- (i) Insufficient facilities to deal with the number of Minimum Cost Quantity batches for a given annual rate of usage. The number of orders required, say, for each bought out component to be ordered in Minimum Cost Quantities may be such that two or three times the existing purchasing office facilities would be required. Although theoretically the provision of sufficient facilities is justified, these facilities may not be made available easily owing to building limitations, local labour conditions, etc. Hence batch quantities larger than the Minimum Cost Quantity will have to be accepted. Similarly, insufficient Production Control, Inspection, and Accounting facilities or skilled operators may limit the number of batches, and hence batch sizes will have to be larger than the Minimum Cost Quantity. Further, existing machine capacity may be insufficient to permit setting up for the number of Minimum Cost batches together with unit manufacturing time, and again a smaller number of set ups only will be possible with larger batch sizes than the Minimum Cost Quantity.

These restrictions should be regarded as short term, as steps should be taken to increase facilities, if the increased costs and continuous demand are such as to justify the facilities being increased.

- (ii) Company with under employed capital. When a company has capital not being used (as distinct from Bank overdrafts) then the additional cost to the company of using this capital is nothing. Hence, under these conditions, the value of (I) will be reduced and a larger Minimum Cost Quantity will result. It is obvious that this condition will only be a temporary one, for if the under-employment of the company's capital is likely to continue for some time, then the under-employed capital would be invested elsewhere.

Thus it can be seen that a batch size will only be above the Minimum Cost Quantity in two special cases, one of a very short term nature, and the other perhaps on a longer term according to the ability of a company to increase the appropriate facilities.

(b) Factors which make the Economic Batch Size below the Minimum Cost Quantity

- (i) Rate of return on capital employed. As can be seen from Table A in Appendix 2, quite a substantial reduction in the Minimum Cost Quantity will cause only a very small increase in the total unit cost. For example, a 20% reduction in quantity from the Minimum

Cost Quantity (i.e. a batch 80% the size of the Minimum Cost Quantity) will cause only a maximum increase of 1.25% in cost.

Now, if the profit on an article is, say, 25% of the Minimum Total Cost, the Minimum Cost Quantity lasts one year, and the average capital invested is half the Minimum Cost Quantity, then the annual rate of return on the capital invested in stock will be :

$$\frac{0.25 \times \text{Minimum Cost} \times \text{Minimum Cost Quantity} \times 100\%}{\text{Minimum Cost} \times \text{Minimum Cost Quantity} \times 0.5} = 50\%$$

However, if the batch size is reduced to 80% of the Minimum Cost Quantity then the minimum profit per article will be 25 - 1.25% or 23.75% the Minimum Total Cost. But the reduced batch will only last 0.8 of the year, and therefore the annual rate of return on capital invested is:

$$\frac{23.75 \times \text{Minimum Cost} \times \text{Minimum Cost Quantity} \times 100\%}{1.0125 \times \text{Minimum Cost} \times 0.8 \text{ Minimum Cost Quantity} \times 0.5}$$

or 58.64% minimum, and this percentage could rise to nearly 62.5%.

Thus a smaller batch size than the Minimum Cost Quantity will provide a bigger profit return on the money invested in stock, and hence this smaller batch size is more economic from an overall company financial view.

However, the situation set out above only takes into account the money invested in stock and ignores the capital invested in machinery and other facilities necessarily required to manufacture one article. For example, consider a company which makes an article for £1 when it makes 4,000 in a batch, this quantity being consumed in one year, and makes the same article for £1. 1. 0. when it makes 2,000 in a batch lasting half a year. The article is sold for £1. 5. 0.

If investment in stock only is considered, then on the same basis as above the rate of return on the 4,000 batch is

$$\frac{5/- \times 4,000}{20 \times 0.5 \times 4,000} \text{ or } 50\%, \text{ and on the 2,000 batch :}$$

$$\frac{4/- \times 2,000}{20 \times 0.5 \times 2,000 \times 1.05 \times \frac{1}{2}} = 76.1\%$$

Under these conditions clearly the 2,000 batch is more profitable.

However, if the capital invested in machinery is included, say £10,000, then the 4,000 batch has a rate of return of

$$\frac{4/- \times 2,000}{20 \times [10,000 + 0.5 \times 2100] \frac{1}{2}} = 7.24\%$$

Thus a different result is obtained if capital invested in plant and facilities is included. Nevertheless, there is a size of batch somewhat smaller than the Minimum Cost Quantity which gives a higher rate of return when all investment is considered, and further research is necessary to find equations for this batch size.

- (ii) Capital Limitations. When a company has insufficient capital to finance the Minimum Cost Quantity then it has to accept a smaller batch. It can be seen from Table A in Appendix 2 that if every batch is reduced to 70% of the Minimum Cost Quantity the resulting rise in total unit cost is only 3.21% maximum, and in practice for the average company the resulting rise varies from 0.5 to about 2.5% for individual components.
- (iii) Flexibility to meet changing market conditions. For certain products the Minimum Cost Quantity may be so large that the time to consume it is longer than the time that the demand for the product is likely to exist. This may well apply to seasonal and fashionable goods, and to items in an industry, with rapid technical development.

It is suggested that the size of these batches is reduced to a quantity that will be consumed shortly before the product is out of fashion or has been superseded. If the reduction in batch size is such that the percentage increase in cost is significant, consideration should be given to adjusting the selling price, or in extreme cases consideration may have to be given as to the advisability of continuing manufacturing and marketing the product.

- (iv) Increase in Work-in-Progress. The size of batches, in certain instances, may be so large as to occupy the available capacity of particular processes for long periods during their manufacture, often several weeks. Other products which may require these particular processes therefore, may be delayed, and hence increase the work-in-progress of the company. This increase may be to a level which more than offsets any corresponding gain in reduction in cost by manufacturing the Minimum Cost Quantity of a particular component or product.

In addition to large batches of a product delaying other products and increasing work-in-progress, if the product takes a long time to manufacture and spends only a comparatively

short time in stores the major holding charges will be incurred during the Work-in-progress period.

Further research is being carried out on both these aspects of the effect of Work-in-progress on Economic Batch Sizes.

- (v) Technical Limitations. The life of a set up in some cases may not last the life of a batch, e.g. the necessity to resharpen tools, with consequent resetting of a machine, and hence increased setting costs may be incurred after a certain point. Thus the size of a batch may be limited to the life of a set up, unless the additional cost of sharpening is so small in relation to total cost of set up that the consequent slight increase in Minimum Cost Quantity is justified.

- (c) Factor which makes the Economic Batch Size either above or below the Minimum Cost Quantity

The quantity of a component, which is incorporated in only one or two final products must be directly related to the quantity of the final product being manufactured, otherwise insufficient or surplus components will be made. It may well be that it will be cheaper to adjust the Minimum Cost Quantity to a multiple or sub-multiple of the final product quantity than to manufacture one or more Minimum Cost Quantity batches, and then to manufacture a smaller supplementary batch to complete the quantity for the final product. If such adjustments are justified, they should be made by manufacturing the quantity which will give the least increase in cost over the Minimum Cost; e.g. if the Minimum Cost Quantity is 100, approximately the same percentage cost increase (max. 3.0%) will occur if the batch size is 71 or 142 (obtained from Table A, Appendix 2). Thus if the final product quantity is 150 and there is one component per final product, then it will be cheaper to make two batches of 75 than one batch of 150; conversely if the final product quantity is 130, probably it will be cheaper to make one batch of 130 than two batches of 65. Further research is required in this direction.

8. SUMMARY OF THE EFFECT OF ALTERING BATCH SIZES AWAY FROM THE MINIMUM COST QUANTITY

Thus, from the foregoing, it is clear that, in general, the batch size which gives the best financial and economic gain is less than the Minimum Cost Quantity, as the consequent increases in total unit cost is small and may be more than offset by corresponding advantages of bigger returns on capital, reduced work-in-progress and greater marketing flexibility.

The exceptions to this general rule that the Economic Batch Size is smaller than Minimum Cost Quantity occur under conditions of insufficient manufacturing or office capacity to deal with increased numbers of batches, and occasionally the 'rounding off' of batch sizes to multiples or sub multiples of final product quantities.

Much research remains to be done to be able to calculate the correct reductions to the Minimum Cost Quantity, or to produce new formulae to take into account reductions in work-in-progress, flexibility for marketing etc.

In the meantime, companies may reduce their batch sizes considerably below the Minimum Cost Quantity provided they are aware of, and can accept the small increase in Unit Total Cost, as shown in Appendix 2.

However, when considering the permissible percentage increase, companies will no doubt realise that an increase of say, 3% on the total cost may have to be set off against say a 10% profit margin limited by competition, etc. Under these conditions the reduction in profit would be 30%.

The capital made available by reducing the size of batches may well be used for other purposes such as improved manufacturing facilities, sales promotion, research, etc.

PART IV. THE ASSESSMENT OF THE VALUES OF U_t , A, I.

9. INTRODUCTION

In order to avoid calculating misleading values for the Minimum Cost Quantity, the right quantities must be given to the terms U_t (unit total cost) A (set up cost) and I (cost of holding charges expressed as a percentage). There appears to be no general agreement as to what these terms should include; and there is considerable doubt as to what is their quantitative value.

This part of the report endeavours to clarify what these terms should include. The first half shows that overheads should be included in U_t and A; and that, if the rate added to U (the unit labour cost) and A is the normal product on-cost rate, the resulting error in the Minimum Cost Quantity produces only a very small and normally acceptable increase in total unit cost (seldom above 1% and generally below $\frac{1}{3}\%$).

The second half deals with the various components of the holding charge I, expressed as a percentage on total manufacturing costs, and suggests methods of assessing the various percentages for interest on capital, insurance, physical deterioration, obsolescence and storage. It further shows that under certain conditions an upper and lower limit to the batch size should be fixed, and if the calculated Minimum Cost Quantity falls outside these limits then the limit value should be the correct batch size.

Section 1. THE INCLUSION OF OVERHEADS IN UNIT TOTAL COST AND SET UP COSTS WHEN CALCULATING MINIMUM COST QUANTITIES.

10. INTRODUCTION

Differing views are held as to whether on-cost rates to cover overhead expenditure should be added to the labour cost for setting up for manufacture, and also to the labour and material cost for manufacturing each unit after set up when using the formula $\frac{200 \text{ AR}}{U_t I}$. Some companies consider that overheads are already accounted for if the costs of issuing paper work, progressing orders, etc. are included in set up costs, and hence an on-cost rate should not be incorporated in the values of A and U_t .

In the following paragraphs overheads are analysed and classified. The theoretically correct overhead as well as the normal product on-cost rate is added to set up and unit manufacturing labour charges, and the resulting Minimum Cost Quantities compared. The cost of the error in batch size is shown to be small.

11. GROUPING OF OVERHEADS

In considering the question whether overheads should be included in the values of A and U_t , it is necessary to consider what items are covered in these overheads. These items can be grouped for the purposes of this investigation into three classes :-

- (a) Overheads which are incurred both during set up and during manufacture of components after set up.

Certain items of overhead cost will be incurred whether the machine (or operator) is engaged in setting up or in manufacturing components after setting up, e.g. cost of machine depreciation, heating of factory, employee insurance, portion of holiday pay, supervision, salaries, etc. The cost of these items are normally recovered on an hourly basis by dividing this total cost by the total number of operating hours during which this expenditure is incurred.

However, as an alternative, the on-cost rate could be based solely on time for manufacturing a component after set up. This arrangement would require

- i. An estimate of the time spent on component manufacture after set-up as a percentage of the total time of employment of the machine or operator, in order to obtain the number of hours over which the expenditure should be spread. This percentage will be affected by the number of batches of components for a given total manufactured quantity. Such an estimate would be difficult to make and could lead to a large over or under recovery of overhead.
- ii. Double the amount of time booking for each batch, i.e. ON-OFF for setting up and ON-OFF for manufacture after set up compared with ON-OFF for total manufacturing time. This increase in time booking would increase the time spent unproductively and increase the expenditure on costing.

Therefore it would appear that the normal method of computing on-cost rate should be adhered to, and under this method it is clear that at least a portion of the overhead cost must be added to both set up and manufacturing costs when calculating batch sizes. This portion is referred to as 'B' and the overhead items in this portion are listed in Appendix 3B. On first analysis this portion appears to be 70 - 85% of all overheads.

- (b) Overheads which are incurred solely in issuing, progressing and completing each manufacturing order.

Certain items of overhead expenditure are incurred solely in issuing, progressing and completing manufacturing orders. The cost of these items bear no relation to time of set up for any individual order, but are related to the number of manufacturing orders. Hence when computing on-cost rates for the purpose of calculating Minimum Cost Quantities, these items strictly should be taken out of overheads and an average cost for these items added to the labour cost for each set up. This portion of the overhead is referred to as 'C' and a list of these overhead items is shown in Appendix 3C. First analysis shows this portion to be about 4 to 14% of all overheads.

A major assumption is made, when giving a rate for costs associated with each order in that these costs will vary directly with numbers of orders. Thus, if orders are reduced or increased, corresponding reductions or increases will be made in staff, stationery, etc. associated with issuing, progressing and completing orders.

- (c) Overheads which are incurred when machine is manufacturing after set up is completed.

Certain items of overhead expenditure are incurred in running the machine (or operator) to manufacture the components after set up is completed. These overheads are incurred on a time basis, e.g. power, grease, etc. This portion of the overhead is referred to as 'D' and a list of the overhead items concerned are listed in Appendix 3D and appears to be about 7 to 17% of all overheads.

12. COMPARISON OF USING DIFFERENT 'ON-COST' RATES IN CALCULATING MINIMUM COST QUANTITIES.

- (a) Necessity for different rates for internally manufactured items and bought out items.

The first analysis of overheads shows that the major portion (70-85%) of these overheads cover items which are incurred whether the machine or operator is setting up or manufacturing items after set up, and the smallest portion (4-12%) of these overheads are for items related to the issuing, progressing and completing of an order. From this preliminary analysis it is clear that overheads in general cannot be neglected in calculating Minimum Cost Quantities for internally manufactured items.

However, where the component is a 'bought out' item the overheads incurred are solely of the C type, and hence in these cases the cost of 'set up' is solely the cost of placing progressing, transporting, receiving and paying for an order.

Thus when computing Economic Manufacturing Quantities and Economic Ordering Quantities the value for 'set up' cost will be different for internally manufactured items and bought out items.

- (b) Correct values of on-cost rate and order cost to be added to labour costs for internally manufactured items.

The correct values of on-cost rate and order costs to be added to labour set up and manufacturing costs are as follows :-

- (i) On-cost rate during set up and Order Cost

$$\text{On-cost rate during set up} = \frac{BH}{T_t}$$

$$\text{Order cost} = \frac{CH}{N}$$

where B is the ratio $\frac{\text{overhead costs in Schedule B}}{\text{Total Overhead Cost}}$

H is the total overheads in the overhead budget period

T_t is the total number of operating hours in the overhead budget period

C is the ratio $\frac{\text{overhead costs in Schedule C}}{\text{Total Overhead Cost}}$

N is the number of orders issued per machine/operator in the overhead budget period

- (ii) On-cost rate during unit manufacturing time

$$\text{On-cost rate} = \frac{BH}{T_t} + \frac{DH}{T_m}$$

where D is $\frac{\text{the ratio overhead costs in Schedule D}}{\text{Total Overhead Costs}}$

T_m is the total number of unit manufacturing hours in the budget period during which the costs in schedule D are incurred

(c) Comparison of using correct on-cost rates, order cost and product costing rates in computing ratio $\frac{A}{U_t}$

(i) Ratio $\frac{A}{U_t}$ using theoretically correct on-cost rates and order cost

$$\frac{A}{U_t} = \frac{T_s (L + \frac{BH}{T_t}) + \frac{CH}{N}}{T_u (L + \frac{BH}{T_t} + \frac{DH}{T_m}) + M} \quad \dots\dots (1)$$

where T_s is the time for set up

T_u is the time for unit manufacture

L is the direct labour cost per hour

M is the component material cost

Now let $T_m = mT_t$ i.e. m is the ratio unit manufacturing time
total operating time

$N = nT_t$ i.e. n is the number of orders issued per
machine/operator per hour

$\frac{H}{T_t} = R$ i.e. R = normal product on-cost rate

Then
$$\frac{A}{U_t} = \frac{T_s (L + BR) + \frac{C}{n} R}{T_u (L + (B + \frac{D}{m})R) + M} \quad \dots\dots (2)$$

(ii) Ratio $\frac{A}{U_t}$ using normal product costing on-cost rate

$$\frac{A}{U_t} = \frac{T_s (L + R)}{T_u (L + R) + M} \quad \dots\dots (3)$$

(iii) Percentage Error in Minimum Cost Quantities using normal product costing on-cost rates in lieu of theoretically correct rates.

$$\% \text{ Error} = \frac{Q_{\text{incorrect}} - Q_{\text{correct}}}{Q_{\text{correct}}} \times 100$$

where Q = Minimum Cost Quantity

$$= \sqrt{\frac{200 AR}{U_t I}}$$

$$= K_1 \sqrt{\frac{A}{U_t}} \text{ where } K_1 = \sqrt{\frac{200 R}{I}}$$

$$\therefore \% \text{ Error} = \frac{K_1 \sqrt{\frac{A}{U}}_{\text{incorrect}} - K_1 \sqrt{\frac{A}{U}}_{\text{correct}}}{K_1 \sqrt{\frac{A}{U}}_{\text{correct}}} \times 100$$

$$= \left(\frac{\frac{A}{U}_{\text{incorrect}}}{\frac{A}{U}_{\text{correct}}} - 1 \right) \times 100$$

$$\therefore \% \text{ Error} = \left(\frac{\frac{T_s (L + R)}{T_u (L + R) + M}}{\frac{T_s (L + BR) + \frac{C_R}{n}}{T_u (L + (B + \frac{D}{m})R) + M}} - 1 \right) \times 100$$

$$= 100 \left(\frac{\frac{L + R}{L + R + \frac{M}{T_u}}}{\frac{L + BR + \frac{CR}{nT_s}}{L + (B + \frac{D}{m})R + \frac{M}{T_u}}} - 1 \right)$$

$$= 100 \left(\frac{L + R}{L + (B + \frac{C}{nT_s})R} \times \frac{L + (B + \frac{D}{m})R + \frac{M}{T_u}}{L + R + \frac{M}{T_u}} - 1 \right)$$

Let $R = YL$ where Y is the ratio of $\frac{\text{overhead cost}}{\text{direct labour cost}}$

$$\therefore \% \text{ Error} = 100 \left(\frac{\frac{L + YL}{L + (B + \frac{C}{nT_s})YL} \times \frac{L + (B + \frac{D}{m})YL + \frac{M}{T_u}}{L + YL + \frac{M}{T_u}} - 1 \right) \times 100$$

$$\therefore \% \text{ Error} = 100 \left(\frac{\frac{1 + Y}{1 + (B + \frac{C}{nT_s})Y} \times \frac{1 + (B + \frac{D}{m})Y + \frac{M}{LT_u}}{1 + Y + \frac{M}{LT_u}} - 1 \right)$$

Now let $K_2 = \frac{M}{LT_u} = \frac{\text{unit material cost}}{\text{unit direct labour manufacturing cost}}$

$$\therefore \% \text{ Error} = 100 \left(\frac{\frac{1 + Y}{1 + (B + \frac{C}{nT_s})Y} \times \frac{1 + (B + \frac{D}{m})Y + K_2}{1 + Y + K_2} - 1 \right)$$

Taking values :- $B = 0.7$
 $C = 0.1$
 $D = 0.15$
 $m = 0.75$
 $Y = 2$
 $n = 0.125$ i.e. One order every 8 hrs/machine
 $T_s = 1$
 $K_2 = 1$

Error = - 15%, i.e. the Minimum Cost Quantity calculated on the basis of using the product costing on-cost rate is 15% smaller than it should be if the theoretically correct on-cost rates were used; and it would appear to be rare that the incorrect economic manufacturing quantity will be more than 20% smaller or 8% larger than the correct quantity when using the proper on-cost rates.

It is suggested that individual companies should calculate their own errors and decide whether such errors are significant, bearing in mind that, owing to the flatness of the cost curve around the Minimum Cost Quantity, very small differences in total Unit Costs occur for fairly large variations from this quantity from Table A Appendix 2. 20% decrease in Minimum Cost Quantity will cause a maximum increase of 1.25% on total Unit Costs and 8% increase in Minimum Cost Quantity will cause a maximum increase of 0.17%.

Section 2. ASSESSMENT OF THE VALUE OF I.

13. COST OF CAPITAL INVESTED IN STOCKS

When considering the percentage on unit manufacturing cost to cover capital interest charges included in the holding charge percentage I, it is necessary to consider the cost to the company of having capital invested in stock. Set up costs, direct labour and material costs are actual monies paid out by the company, and so, therefore, must be the cost of interest on capital invested in stocks.

Where stocks have been financed by increasing the company's Bank overdraft, then the cost to the company is the Bank interest rate multiplied by the capital invested.

However where the capital has been found from internal resources, the company must decide what it has cost to use this money. This cost may vary from nothing when the company leaves capital idle if such capital is sufficient to cover the investment in stocks in question, through Gilt Edged investment rate when the company invests its surplus temporarily in Gilt Edged stocks, to loss of earning power if invested in known alternative and available production programmes and facilities.

This rate of interest can only be decided by the company concerned knowing its future capital investment plans over the period in question.

14. COST OF INSURANCE

This is a straight percentage on the value of the stock and is normally very small.

15. COST OF STORAGE - INTRODUCTION

When considering the cost of storage in calculating Minimum Cost Quantities this cost is solely the rent, rates, maintenance, heating and lighting of the space occupied. The cost of any labour associated with the receipt and issue of stores is not included in the cost of storage, as receipts are associated with set up costs, and issues depend on the batch size of the next stage of assembly or rate of use. These labour costs have been included in overheads.

The provision of storage facilities can be of three types :-

- (a) Fixed storage space which is provided whatever the batch quantity and whether any stock is held or not, e.g. bins in racks allocated to specific items.
- (b) Storage space allocated to a product in proportion to the batch size, this space remaining allocated to that product until the batch is exhausted, the area then being allocated for other purposes, e.g. items placed in marked off areas of a store, or storage space for a product specially rented over and above normal fixed storage facilities.
- (c) Storage space which reduces as the batch is consumed, e.g. items stored in a stack of standard pallets which, as pallets are emptied, these pallets are used for other items and returned to the pallet stack.

16. CALCULATION OF MINIMUM COST QUANTITIES UNDER VARYING STORAGE CONDITIONS

- (a) Calculation of Minimum Cost Quantity when storage space is fixed irrespective of quantity

Where storage space is provided, irrespective of size of batch, no additional storage costs are incurred by increasing the size of a batch quantity. In these cases the percentage on total unit costs for storage can be ignored when calculating Minimum Cost Quantities.

- (b) Calculation of Minimum Cost Quantity when storage space is allocated in proportion to batch quantity and remains allocated until batch is exhausted

Let S be the unit storage charge per year.

Then let total cost for batch quantity $Q_1 = V_1$

$$V_1 = (A + U_t Q_1) \left(1 + \frac{I Q_1}{200 R}\right) + S Q_1 \times \frac{Q_1}{R}$$

$$V_1 = A + U_t Q_1 + \frac{A I Q_1 + U_t I Q_1^2}{200 R} + \frac{S Q_1^2}{R}$$

$$\therefore \text{Unit cost} = \frac{A}{Q_1} + U_t + \frac{A I}{200 R} + \frac{U_t I Q_1 + 200 S Q_1}{200 R}$$

$$\therefore \frac{d \text{ cost}}{d Q_1} = \frac{A}{Q_1^2} + \frac{U_t I + 200 S}{200 R}$$

$$\text{For minimum } \frac{d(\text{cost})}{d Q_1} = 0$$

$$\therefore Q_1 = \sqrt{\frac{200 A R}{U_t I + 200 S}} \quad \dots (1)$$

- (c) Calculation of Minimum Cost Quantity when storage space is allocated in proportion to batch quantity but reduces as batch is consumed

As before let S be the unit storage charge per year.

Then let total cost for batch quantity $Q = V_2$

$$V_2 = (A + U_t Q_2) \left(I + \frac{I Q_2}{200R} \right) + S Q_2 \times \frac{Q_2}{2R}$$

$$\text{Similarly } Q_2 = \sqrt{\frac{200AR}{U_t I + 100S}} \quad \dots (2)$$

- (d) Simplification of formulae 1 and 2

$$\text{Let } I_s \text{ be } \frac{S}{\text{unit cost}} \times 100\%$$

$$\therefore I_s = \frac{S}{\frac{A}{Q} + U_t} \times 100 \quad \dots (3)$$

If $\frac{A}{Q}$ is small in relation to U_t and can be neglected then Equation 3

$$\text{becomes } S = \frac{U_t I_s}{100}$$

and equations 1 and 2 become respectively

$$Q_1 = \sqrt{\frac{200AR}{U_t(I + 2I_s)}}$$

$$Q_2 = \sqrt{\frac{200AR}{U_t(I + I_s)}}$$

Thus in certain cases a percentage rate for storing items can be added to the other holding charges.

17. STORAGE DURING MANUFACTURE

Certain batches are restricted in size owing to shop floor storage facilities being insufficient to accommodate a batch or would cause undue congestion. In these cases a limit must be placed on the batch size, this limit being independent of storage costs.

18. RESEARCH INTO THE VALUES OF STORAGE COSTS

It is suggested that a research is made into

- (a) the division of products into the three types of storage costs
- (b) the value of S
- (c) how often $\frac{A}{Q}$ can be neglected and hence storage costs can be treated as a direct percentage to be added to other holding charges.

19. COST OF PHYSICAL DETERIORATION OF STOCK - INTRODUCTION

Deterioration of stock is the rendering unusable of a product through evaporation, oxidisation, or other physical or chemical change.

Products vary widely in their susceptibility to deterioration and can be classified accordingly.

20. CLASSIFICATION OF DETERIORATION OF COMPONENTS

(a) No Deterioration

Certain items will not deteriorate at all, or only after a period several times longer than the life of a batch. Such items should not bear any percentage of total batch cost for depreciation.

(b) Constant rate of Deterioration

Some items deteriorate at a constant rate, e.g. evaporation of oil held in storage tanks. These items should bear a constant percentage of total batch cost for deterioration which may be added to the holding charges.

(c) Deterioration causing Rejection after a Given Period of Time

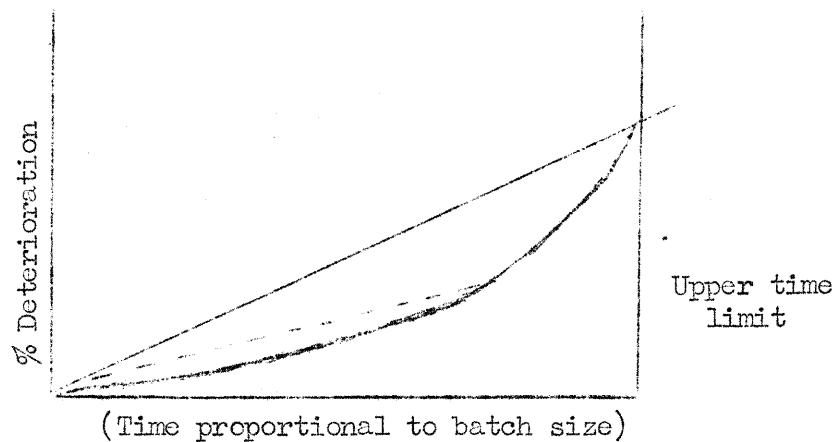
Other items deteriorate slowly, often not noticeably, and insufficiently to cause their rejection. However, after a period, a second stage is reached where the deterioration is enough to cause the rejection of some of the items, and in a short period the whole of the items remaining in a batch will be rejected, through deterioration.

It is clear that for these items no deterioration charge can be applied to the early period, but the size of the batch must be limited so that under normal consumption rates there will be no items remaining in stock at the start of the second period.

Thus for these items no percentage of total cost for deterioration should be added to the holding charges but an upper limit to the batch quantity must be imposed.

(d) Deterioration rate altering with time

Products like perfume, fine pharmaceuticals, etc., may deteriorate slowly at first, and then deteriorate more rapidly with increase in time. It is difficult to apply a simple calculable formula to provide for the deterioration of these products, and it is suggested that a constant rate for deterioration is included in the holding charges percentage rate I with an upper limit to the batch size.



If the time to consume the calculated Minimum Cost Quantity is shorter than the time to reach the upper time limit, then the correct percentage rate for deterioration (represented by the slope of the dotted line on the graph) which should have been applied is smaller than the actual rate (represented by the slope of the full line) applied in the formula. Thus by applying this larger rate a smaller batch size has been obtained than should have been the case. Investigation in individual companies would show whether this reduction in size is significant.

21. SUMMARY OF METHODS OF ACCOUNTING FOR DETERIORATION

It will be seen that deterioration can be accounted for as set out in the table below :-

	<u>Percentage added to holding charges</u>	<u>Upper limit imposed</u>
a. No deterioration	No percentage applied	No limit
b. Constant rate of deterioration	Percentage applied	No limit
c. Deterioration causing rejection after a period of time	No percentage applied	Limit applied
d. Deterioration rate altering with time	Percentage applied	Limit applied

Thus deterioration can be covered by adding a percentage to holding charges and imposing an upper limit in certain cases.

22. THE COST OF OBSOLESCENCE OF A BATCH - INTRODUCTION

A cost has to be allowed for a batch becoming obsolescent due to any remaining products in that batch no longer being required through an unforeseen fall in demand of the product or supersession by another product.

It would appear that this cost can be estimated only after consideration of two factors :

- (a) The possibility of a batch of the product (or portion thereof) becoming obsolescent
- (b) The cost of disposing of the remaining products of an obsolescent batch.

23. THE POSSIBILITY OF A BATCH BECOMING OBSOLESCE

A batch, or portion thereof, will only become obsolescent when the demand falls unexpectedly, or technical developments are unknown to the stock planner at the time of ordering the batch.

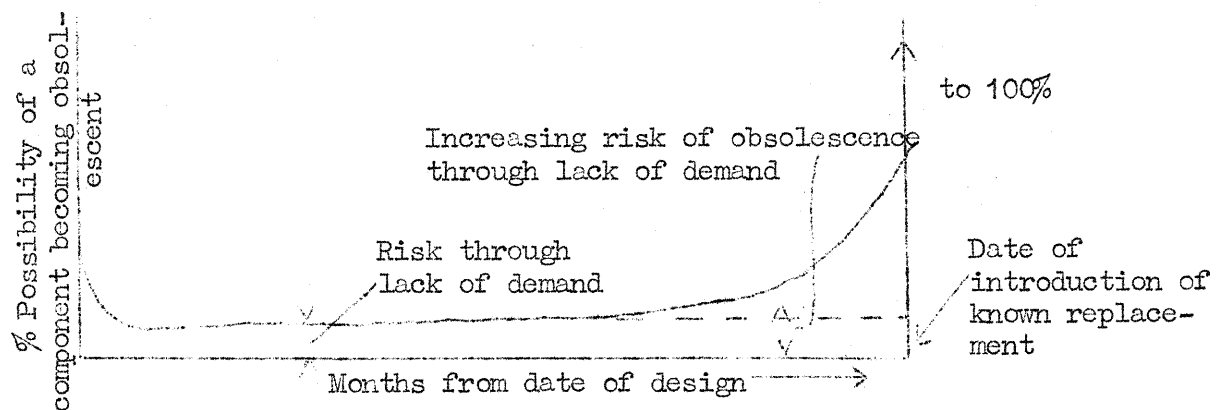
If it is known that a product will be replaced in a given time, then obsolescence should not occur if the size of the batch is calculated to be consumed by that date.

It would appear reasonable to assume that the longer the time the product has been in service the larger is the risk of obsolescence occurring

(with the probable exception of highly standardised components e.g. nuts and bolts).

However, it is possible that a higher risk of obsolescence may occur immediately after the design period until the component is proved in service.

Thus the possibility of a batch becoming obsolescent would appear to be given by a curve of the shape:



Obviously no quantities or scales can be applied to the axes of such a graph without considerable research, and special consideration would have to be given to seasonable and short life fashion and similar goods. Industries with a high development rate will have in general components with a shorter x axis than components in a more stabilised development industry.

As a corollary it is clear that the larger the batch and the longer the life of that batch, the larger will be the risk of obsolescence occurring during the life of that batch.

24. ALLOWING FOR OBsolescence WHEN CALCULATING MINIMUM COST QUANTITIES

Normally a product should become obsolescent only once during its life, i.e. at its end, and therefore an actual cost of a product becoming obsolescent will only occur in one batch.

A reduction in the size of each batch will reduce the cost of obsolescence when it does occur. However if one applies this reduction to each batch of the product then the total cost of each unit of product will be increased above the minimum cost as shown in Appendix 2. Thus the total saving in reducing the cost of disposing of the remaining products of an obsolescent batch must not exceed

$$n \times Q \text{ times } \frac{A}{Q} \frac{(x-1)^2}{x} \quad \text{where } n \text{ is the total number of batches of quantity } xQ \text{ produced since the design of the product.}$$

$$= nA(x-1)^2$$

When a component becomes obsolescent three methods exist for disposing of the remaining products:

- (a) The remaining products may be scrapped
- (b) The remaining products may be consumed but at a slower rate
- (c) The remaining products may be used at the same rate but the value of the product may be the same or less.

It is not yet clear how the reduction in batch size, the various methods of disposal and the possibility of obsolescence occurring are related. Basic, as well as quantitative, research is required to determine the method of allowing for obsolescence when calculating the Minimum Cost Quantity. It is suggested that for the time being the practice is continued of including a small percentage for obsolescence in the holding charges percentage rate I, together with stating a date after which components should not be held in stock.

25. SUMMARY OF PERCENTAGES TO BE CONSIDERED IN OBTAINING THE PERCENTAGE VALUE OF I FOR TOTAL STOCK HOLDING CHARGES

It has been seen that the percentage to be applied for I the holding charges in the Minimum Cost Batch formula is made up of a :

- (a) Rate for insurance
- (b) Rate for storage, with upper limit to size of batch imposed if necessary
- (c) Rate for deterioration, with upper limit to size of batch imposed if necessary
- (d) Rate for obsolescence, which increases with time, occasionally with date beyond which stocks should not remain
- (e) Rate for capital interest charges.

The first three percentages will not change during the life of the design of the product but the latter two may alter. It is suggested that a, b and c are amalgamated and stated on the stock card together with the lower of the two upper limits when necessary.

PART V. METHODS OF OBTAINING THE MOST SUITABLE ECONOMIC BATCH QUANTITY

26. STAGES IN CALCULATING AN ECONOMIC BATCH SIZE

From the foregoing parts of this report, and the present state of knowledge, the steps in obtaining the most suitable economic batch size for a product or component appear to be :-

1. Calculate the Minimum Cost Quantity
2. Calculate the maximum reduction of the Minimum Cost Quantity which has an acceptable cost increase, if the company is endeavouring to reduce its capital invested in stock, or wishes to reduce its Work-in-Progress or requires greater flexibility of stock holding
3. Check that the final calculated quantity is within any limits of batch size imposed for the product; and if outside the limit apply that limit.

27. EFFECT OF AVAILABLE MANUFACTURING AND OFFICE FACILITIES ON BATCH SIZE

It has been shown that existing facilities may restrict the number of batches that can be ordered for any given total quantity of output. Thus, before installing any method for ordering Minimum Cost Quantities or suitable Economic Batch Sizes, it is advisable to calculate the estimated increase or decrease on the present number of batches which will occur by the introduction of calculated Minimum Cost Quantities or Economic Batch Sizes.

It is suggested that a representative sample of components is taken and Minimum Cost Quantities calculated for each. Hence for the same output of the sample components, the number of batches required for calculated Minimum Cost Quantities can be compared with the existing number of batches and a ratio of reduction or increase on present orders obtained. Hence any necessary alterations in facilities can be planned. If no alterations are possible, or during the alteration period, if necessary, general adjustments may be made to all batch sizes e.g. all batches should be half (twice etc.) the calculated Minimum Cost Quantity.

28. INFORMATION REQUIRED TO CALCULATE ECONOMIC BATCH QUANTITIES

As has been seen, the factors required to calculate Minimum Cost Quantities, and the denominator in equation 6 Appendix 2 to obtain a reduction in batch size for a given permissible percentage of increase in total unit cost are :

A The set up cost plus overheads

U_t The unit manufacturing cost, manufacturing labour overheads and material

I The holding charges expressed as a percentage of the set up cost and unit manufacturing costs

R The annual consumption of the stock

The component or product stock card should therefore have a section on which A , U_t and I are entered; and as $\frac{A}{U_t}$ will not alter during the life of the component as long as no major alteration to overhead rates occur, $\frac{A}{U_t}$ can also be stated, thereby reducing permanently all subsequent calculations of Minimum Cost Quantities by one step.

Further, any upper or lower limits should be entered on the stock card with the appropriate reason,

- e.g. Storage limitation
- Deterioration limitation
- Work-in-Progress limitation

29. METHODS OF CALCULATING MINIMUM COST QUANTITIES AND ECONOMIC BATCH QUANTITIES

There are four main methods of obtaining Batch Quantities :-

- a. Direct Calculation
- b. Use of Nomographs
- c. Use of Tables
- d. Use of Graph

a. Direct Calculation

All batch quantities can be calculated direct using logarithms tables, or slide rules. However this method is slow and generally beyond the capabilities of normal stock clerks.

b. Use of Nomograms

Nomograms can be used to evaluate Q and from this the value of

$K = 2 + \sqrt{\frac{200R U_t}{IA}} + \sqrt{\frac{IA}{200R U_t}}$ can be obtained (the denominator in the equation $P = \frac{(x-1)^2}{\frac{x}{K}}$ giving the increase percentage in cost for a given reduction in size of the Minimum Cost Quantity).

No fewer calculating steps will be made if a separate nomogram is provided for each value of I . The total number of nomograms necessary will be of the order of 50 at the most, if values of I are increased in integers. A maximum 8 to 10 nomograms will satisfy most companies.

However, in order to simplify the nomogram for estimating Q, a nomogram has been designed to give an estimate of the value of the order, i.e. $Q U_t$ called Q_v

$$\text{Thus } Q_v = Q \times U_t = \sqrt{\frac{200AR \times U_t^2}{I U_t}} = \sqrt{\frac{200A}{I}} \times \sqrt{\text{Value of Annual Consumption}}$$

$$\text{Let Value of annual consumption} = \frac{V_R}{U_t} \\ \text{and } m = \sqrt{\frac{200A}{I}}$$

This nomogram is shown in Appendix 4.

Thus Q_v is estimated by calculating m and V_R and joining these points on their respective scales. The value at which this straight cuts the Q_v scale gives the required value.

$$\text{The order quantity } Q \text{ can then be obtained by } Q = \frac{Q_v}{U_t}.$$

If it is required to reduce the batch size below the Minimum Cost Quantity, then a second nomograph can be used which, for various values of K, will give the reduction in batch size x for a given permissible percentage increase in cost.

The Economic Batch size can thus be obtained by multiplying the Minimum Cost Quantity by the appropriate factor x and subsequently checking on the stock card that no limits have been exceeded.

c. Use of Tables

Tables can be used for the determination of both Minimum Cost Quantity and the reduction in stock for a given permissible increase in cost. In addition, it is possible also to show for any Minimum Cost Quantity the time to consume that quantity.

Normally tables are designed to obtain one value from cross referring to two others. Thus, to obtain the required batch size, it will be necessary to take four steps:

Step 1. From tables of $\frac{A}{U_t}$ against I the value $\frac{200A}{U_t I}$ can be obtained.

Step 2. From tables of $\frac{200A}{U_t I}$ against R the Minimum Cost Quantity can be obtained, and as each Quantity in the table is associated with a rate of usage R then the consumption time of that quantity can also be shown.

Step 3. From tables of Q against $\frac{A}{U_t}$ the value of $K = (2 + \frac{QU_t}{A} + \frac{A}{QU_t})$ can be found.

Step 4. From tables of K and x, the reduction in size of the Minimum Cost Quantity, the percentage increase in cost can be obtained.

However, if separate tables are available for each value of I, by cross referring to $\frac{A}{U_t}$ and R the three values, Minimum Cost Quantity $\frac{200AR}{U_t I}$, the value $K = \frac{QU_t}{A} + \frac{A}{QU_t}$ and the consumption period of the Minimum Cost Quantity can be found in one step.

A second step (Step 4 above) will now give the reduction in size of the Minimum Cost Quantity x for a percentage increase in cost.

$$I = 20$$

$\frac{A}{U_t}$	R	100	200	300 etc.
10	Q	100	141	
	K	12.1	16.21	etc
	Consumption Period	12 m	8.5 m	
8	Q	89	126	
	K	13.7	17.87	etc
	Consumption Period	10.7 m	7.4 m	
etc	etc	etc	etc	

d. Use of Graphs

Graphs can be used for calculating the Minimum Cost Quantity using Log Log graph paper, one graph for each value of I , with values of Q on the y axis and values of R on the x axis, together with superimposed lines for values of $\frac{A}{U_t}$. Appendix 5a.

The value of K can also be obtained by plotting K against Q , again using superimposed lines for values of $\frac{A}{U_t}$. Appendix 5b.

The percentage increase in cost P for proportional alterations x on the Minimum Cost Quantity can then be obtained by plotting P against x and using superimposed lines for values of K . Appendix 5c.

Thus when using graphs three separate sheets are required, against two sheets for nomographs, and the intersection of three lines is required to be read, including one which has probably to be interpolated.

It would appear therefore that nomograms are preferable to using graphs for the calculation of economic batch sizes.

PART VI. SUMMARY OF FURTHER RESEARCH REQUIRED

30. BASIC RESEARCH

Further research is required to establish principles, and simple equations or relationships to take into account :-

- a. Rate of return on capital including fixed assets.
- b. Effect of work-in-progress.
- c. Adjusting batch sizes of components to multiples or sub-multiples of the final product.
- d. Need for market flexibility.
- e. Obsolescence factor.

31. QUANTITATIVE RESEARCH

Research is required to establish values of holding charges for :-

- a. Various types of storage.
- b. Various types of deterioration.

APPENDIX 1.

COVERING LETTER INTRODUCING THE RESEARCH PROJECT

It would appear from recent discussions that an increasing number of firms are considering the possibility of calculating the optimum Economic Manufacturing Quantity to be ordered when replenishing stocks of components and sub-assemblies.

In an endeavour to assist British industry and, in particular, such firms who are considering using calculated Economic Manufacturing Quantities, it would be of value to discover whether there are any serious limitations to the use of this technique, and, if so, what type of limitation and in which kind of industry.

It would, therefore, be much appreciated if you would assist in this research by completing the enclosed forms and returning one of them to the Department of Economics and Production of this College. Two copies are enclosed in order that you may retain one copy for record purposes.

The results of this investigation will be published and will be available for general circulation. However, the name of any firm supplying data will not be directly associated with any specific data, though we should like to acknowledge the assistance received by publishing a list of firms who have co-operated in this project.

RESEARCH PROJECT into The Extent to which Calculations based on Estimated Values are and could be used in Establishing the most Economic Manufacturing Quantity.

It will be appreciated that there must be one best quantity for the size of a batch of parts which should be manufactured for stock. A very large quantity would have the advantage of distributing the setting up charges over a large quantity of parts, but if the batch is too large the cost of storing the parts for a long period of time will become excessive.

The calculation for the quantity with the minimum unit cost, or Economic Manufacturing Quantity which could be manufactured is given by the following formula, proof of which is given in Appendix A.

$$\text{The most Economic Manufacturing Quantity } Q = \sqrt{\frac{200AR}{UI}}$$

where A = Setting up costs with overheads and cost of order, movement and paper work.

U = Total manufacturing cost per part comprising material, plus labour plus overheads.

I = The rate as a percentage of the average value of the parts in stores which it is decided should be charged to cover interest on capital, plus storage charges, plus an allowance for possible depreciation and obsolescence.

R = Estimated number of parts used per annum.

The use of the above formula in a particular case would be as given in the following two examples :-

	<u>Example A</u>	<u>Example B</u>
A = Setting up costs	£15. 0. 0.	£20. 0. 0.
U = Total manufacturing cost per part	2. 0. 0.	5. 0. 0.
I = Rate of interest on material in stores	20%	10%
R = Estimated number used per annum	300	20

Example A

$$Q = \sqrt{\frac{200AR}{UI}} = \sqrt{\frac{200 \times 15 \times 300}{2 \times 20}} = \sqrt{22500} = 150$$

Example B

$$Q = \sqrt{\frac{200AR}{UI}} = \sqrt{\frac{200 \times 20 \times 20}{5 \times 10}} = \sqrt{1600} = 40$$

The above calculations are very simple and there is little doubt that the estimates for the values of A, U, I and R can be made with greater accuracy than any planning engineer can be expected to estimate the most economic batch quantity by mentally taking these factors into account and giving each of them their appropriate significance. In spite of this, it is thought that the above simple procedure is not used extensively and the present questionnaire is being sent to a number of firms representing a wide range of industries in the hope that some further data can be collected on this problem which will be of assistance to Industry generally.

We should therefore, appreciate your assistance in answering the attached questionnaire. In return, we will undertake to send to you a report on our findings from this enquiry and our recommendations.

APPENDIX A

Let Q = Economic Batch Quantity

A = Setting up costs plus overheads

U = Total manufacturing cost per part comprising material plus labour plus overheads

I = The rate as a percentage of the average value of the parts in stores which it is decided should be charged to cover interest on capital, plus storage charges, plus an allowance for possible depreciation and obsolescence

R = Estimated number of parts used per annum

Manufacturing cost of Q parts = Setting up charge + manufacturing cost per part x number of parts in batch

$$= A + U \times Q$$

Storage charge for Q parts = Capital x Rate x Time

$$= \frac{\text{Manufacturing cost of Q parts}}{2} \times \% \text{ charge} \times \text{Period}$$

$$= \frac{A + UQ}{2} \times \frac{I}{100} \times \frac{Q}{R}$$

Total cost including storage charge

$$= A + UQ + \left(\frac{A + UQ}{2} \right) \times \frac{I}{100} \times \frac{Q}{R}$$

$$= A + UQ + \frac{AIQ}{200R} + \frac{UQIQ}{200R}$$

Cost of one part

$$y = \frac{A}{Q} + \frac{UQ}{Q} + \frac{AIQ}{200RQ} + \frac{UQIQ}{200RQ}$$

$$y = \frac{A}{Q} + U + \frac{AI}{200R} + \frac{UQI}{200R}$$

$$y = A Q^{-1} + U + \frac{AI}{200R} + \frac{UIQ}{200R}$$

For minimum value of y the slope of the curve must be zero. To find the slope of the curve differentiate with respect to Q. $\left(\frac{dy}{dQ} \right)$.

$$\frac{dy}{dQ} = -AQ^{-2} + 0 + 0 + \frac{UI}{200R}$$

For minimum value

$$\frac{dy}{dQ} = \text{zero}$$

Therefore,

$$AQ^{-2} = \frac{UI}{200R}$$

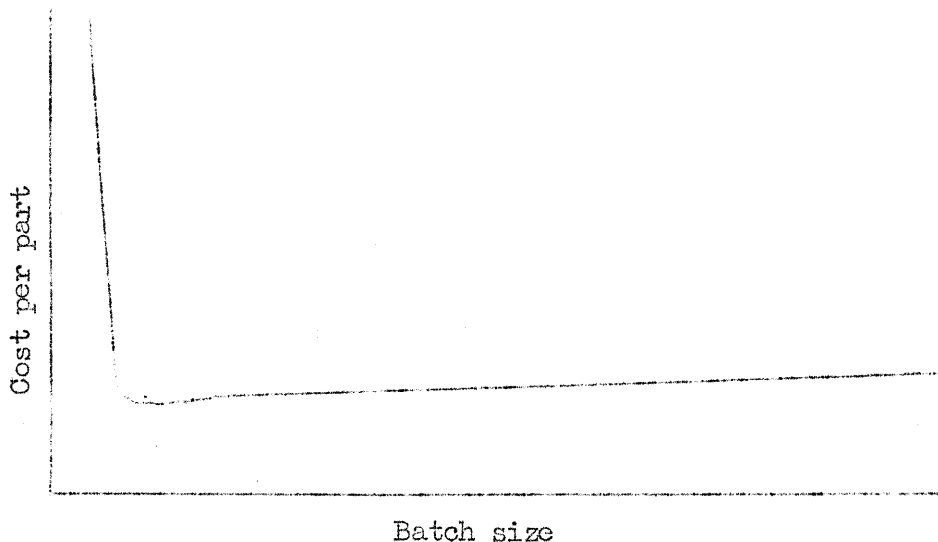
$$\frac{A}{Q^2} = \frac{UI}{200R}$$

$$UIQ^2 = 200RA$$

$$Q^2 = \frac{200RA}{UI}$$

$$Q = \sqrt{\frac{200RA}{UI}}$$

When the cost per part is calculated in accordance with the above formula and the cost per part plotted against the number of parts in the batch, a curve of the type shown below is obtained, indicating a rapid fall in cost during the period when the setting up charges are spread over a larger quantity and after falling to a minimum value, a smaller gradual increase in cost due to the interest charges on the value of material in store.



RESEARCH PROJECT - The Use of Formulae for Calculating
Economic Manufacturing Quantities.

Name of Company:

Industrial Classifications

Please tick the appropriate classification for your Company.

Shipbuilding
Agricultural machinery
Machine tools
Portable power tools
Office machinery
Power generating plant
Mechanical handling equipment and excavators
Other machinery
Clocks, watches, cameras and other mechanical instruments
Electrical instruments
Electrical appliances, (E.g. radio, washing machines, etc.)
Motor cars, motor cycles and parts
Locomotives, railway equipment and rolling stock
Aircraft
Leather goods
Metal goods (e.g. Holloware, cutlery, tools and other metal goods
not otherwise specified)
Textiles
Clothing
Furniture, wood
Pottery, china, glass, etc.
Paper and printing
Other industries

Normal Quantity in batch of finished product

Please tick appropriate classification for the quantity in the normal
batch of finished products.

Unit
Small batch 2 - 5 units per batch
Medium " 6 - 20 units per batch
Large " 21 - 50 units per batch
Very large batch - 51 units and over per batch

Use of Formulae

1. Do you use any method of calculating economic manufacturing quantities ? If so, will you please send details of your scheme, together with examples of values used and the resulting Economic Manufacturing Quantities.
2. If you do not use any method of calculation, would you please use the formula given below choosing about ten typical jobs which cover your range of values for A, U, I, R, recording these values of A, U, I, R together with the resulting Economic Manufacturing Quantities. In addition, would you tell us :-
 - (a) What is the difference between the calculated value for batch size and the one you now use ?
 - (b) Do you think it would be an advantage to change from your present batch size to the one given by the formula ?
 - (c) If you do not support changing your present Economic Manufacturing Quantities to the calculated value, what are your reasons for not wishing to use the value given by calculation.
 - e.g. (i) Size of batch too large, resulting in excessively long runs on a machine thereby tying up existing productive capacity, delaying other work and increasing work-in-progress.
 - (ii) Size of batch too small, resulting in too many batches for the same production causing increase in supervision costs, inspection, progress control, etc.
 - (iii) Size of batch will provide too big a stock causing storage difficulties or running the risk of obsolescence.
 - (iv) Any other reason.
3. In view of these other factors affecting Economic Manufacturing Quantities, do you think a more accurate but inevitably a more complicated formula is necessary ? If so, what additional factors do you think should be included in the calculation ?
4. Do you see any difficulties in using the above formula for the calculation of batch sizes in your organisation and, if so, to what extent do you consider the method unsuitable from the standpoint of :-

- (a) Difficulty in assessing the values for A, R, U, and I for the many parts you manufacture?
- (b) Difficulty in using the formula which involves multiplication, division and the finding of the square root ?

If simple tables, nomograms or cheap, easily operated devices were available, would you utilise them ?

- (c) Any other reason ?

Formula

$$\text{Economic Manufacturing Quantity } Q = \sqrt{\frac{200AR}{UI}}$$

where

- A = Setting up costs with overheads and cost of order, movement and paper work.
- U = Total manufacturing cost per part comprising material, plus labour plus overheads.
- I = The rate as a percentage of the average value of the parts in stores which it is decided should be charged to cover interest on capital, plus storage charges plus an allowance for possible depreciation and obsolescence.
- R = Estimated number of parts used per annum.

FIRMS AND ORGANISATIONS PROVIDING DATA, INFORMATION, SUGGESTIONS AND INTEREST
IN THE RESEARCH PROJECT ON ECONOMIC BATCH SIZES

Accles and Pollock Ltd.
Baker Perkins Ltd.
Boulton Paul Aircraft Ltd.
Bristol Aircraft Ltd.
British Thomson Houston Co. Ltd.
De Havilland Propellers Ltd.
Furniture Development Council
Glacier Metal Co. Ltd.
Glaxo Laboratories Ltd.
Gomme (E.), Ltd.
Hepworth and Grandage Ltd.
Metropolitan Vickers Electrical Co. Ltd.
Rotol Ltd.
Ruston-Bucyrus Ltd.
Shelvoke & Drewry Ltd.
The United Steel Companies Ltd.

APPENDIX 2.

EFFECT ON UNIT COSTS THROUGH ALTERING THE BATCH SIZE FROM THE MINIMUM QUANTITY

Let the ratio of the size of a reduced batch to the Minimum Cost Quantity be x .

Then the increase in unit cost is

$$\begin{aligned} & \frac{A}{xQ} + U_t + \frac{AI}{200R} + \frac{U_t I x Q}{200R} - \left(\frac{A}{Q} + U_t + \frac{AI}{200R} + \frac{U_t I Q}{200R} \right) \\ &= \frac{A}{Q} \left(\frac{1}{x} - 1 \right) + \frac{U_t I Q}{200R} (x - 1) \end{aligned} \quad \dots (1)$$

Now when Q = minimum cost quantity

$$\begin{aligned} Q^2 &= \frac{200AR}{U_t I} \\ \therefore Q^2 U_t I &= 200AR \\ \therefore \frac{U_t I Q}{200R} &= \frac{A}{Q} \end{aligned} \quad \dots (2)$$

Substituting $\frac{A}{Q}$ for $\frac{U_t I Q}{200R}$ in equation (1)

$$\begin{aligned} \text{Increase in unit cost} &= \frac{A}{Q} \left(x - 2 + \frac{1}{x} \right) \\ &= \frac{A}{Q} \frac{(x - 1)^2}{x} \end{aligned} \quad \dots (3)$$

Let percentage increase in Total Unit Cost over Minimum Cost = P

$$P = \frac{\frac{A}{Q} \frac{(x - 1)^2}{x}}{\frac{A}{Q} + U_t + \frac{AI}{200R} + \frac{U_t I Q}{200R}}$$

Dividing through by $\frac{A}{Q} = \frac{U_t I Q}{200R}$

$$\therefore P = \frac{\frac{(x - 1)^2}{x}}{1 + \frac{QU_t}{A} + \frac{IQ}{200R} + 1}$$

$$\text{Now } \frac{A}{Q} = \frac{U_t I Q}{200R} \quad \therefore \quad \frac{I Q}{200R} = \frac{A}{Q U_t}$$

$$\therefore P = \frac{\frac{(x-1)^2}{x}}{2 + \frac{Q U_t}{A} + \frac{A}{Q U_t}} \quad \dots (4)$$

$$\therefore P \left(2 + \frac{Q U_t}{A} + \frac{A}{Q U_t} \right) x = x^2 - 2x + 1$$

$$\text{Let } 2 + \frac{Q U_t}{A} + \frac{A}{Q U_t} = K$$

$$\therefore x^2 - (2 + KP)x + 1 = 0$$

$$\therefore x = \frac{2 + KP \pm \sqrt{(2 + KP)^2 - 4}}{2}$$

This equation gives the largest possible reduction in size without exceeding a stated maximum increase in cost. (The value of x in excess of 1 shows how large the batch can be increased in size without exceeding the stated maximum increase in cost. This value has no useful purpose and can be ignored.) Thus the equation for x is reduced to

$$x = \frac{2 + KP - \sqrt{(2 + KP)^2 - 4}}{2} \quad \dots (5)$$

$$\text{Now } Q = \sqrt{\frac{200AR}{U_t I}} \quad \text{and substituting for } Q \text{ in equation (4)}$$

$$\therefore P = \frac{\frac{(x-1)^2}{x}}{2 + \sqrt{\frac{200R U_t}{I A}} + \sqrt{\frac{I A}{200R U_t}}} \times 100 \quad \dots (6)$$

In the formula
$$P = \frac{\frac{(x-1)^2}{x}}{2 + \frac{t}{A} + \frac{A}{QU_t}} \times 100$$

The denominator cannot be less than 4. i.e. $\frac{QU_t}{A} = 1$

TABLE A

∴ when	x = 0.1	P < 202.5%	x = 1.25	P < 1.25%
	= 0.2	P < 80%	= 1.5	P < 4.13%
	= 0.3	P < 40.75%	= 1.75	P < 8.03%
	= 0.4	P < 22.5%	= 2.0	P < 12.5%
	= 0.5	P < 12.5%	= 3.0	P < 33.3%
	= 0.6	P < 6.67%	= 4.0	P < 56.25%
	= 0.7	P < 3.21%	= 5.0	P < 80.0%
	= 0.8	P < 1.25%	= 6.0	P < 104.2%
	= 0.9	P < 0.27%	= 7.0	P < 128.6%
	= 1.0	P = 0%	= 8.0	P < 153.1%

APPENDIX 3B

OVERHEAD ITEMS WHICH ARE INCURRED WHETHER MACHINE OR OPERATOR IS SETTING
UP OR PRODUCING COMPONENTS AFTER SETTING UP

Works Managers' basic salary

Departmental supervision salaries

Workmen's compensation insurance

General labouring (other than progress labouring)

Process planners	}	Where not recovered by a development expenditure
Work Study		account chargeable to specific products
Tool Designers		

Idle time

Overtime

Training allowance

Holiday pay

Sick pay

Factory rent and rates

Machinery depreciation

Insurance of factory building and plant

Repairs to factory building

Works telephones

Factory lighting and heating

Canteen

Stock control clerks and stock storemen (the portion concerned with
requisitions and issues in
detail to assemblies)

On first analysis it would appear that these costs represent 70 to
85% of overheads in engineering and fabricating industries.

APPENDIX 3C

OVERHEAD ITEMS WHICH ARE INCURRED SOLELY IN ISSUING, PROGRESSING,
SETTING UP AND COMPLETING EACH MANUFACTURING ORDER

Works stationery

Inspection (where 'first off' inspection applies)

Progress clerks (plus their own overheads, furniture depreciation etc.)

Stock control clerks and stock storemen (portion concerned with
placing manufacturing orders and recording receipt
of manufactured items)

Order preparation clerks

Order costing

Labour bonus computations costs

Tool room storemen

Setting up labour costs

Scrapped materials (part concerned with 'first off')

Power for machines (part concerned with 'first off')

On first analysis it would appear that these costs represent 4 to
12% of overheads in engineering and fabricating industries.

APPENDIX 3D

OVERHEAD ITEMS WHICH ARE INCURRED DURING UNIT MANUFACTURE AFTER SET-UP

Small value production materials

Oils, greases, waste, etc.

Consumable tools

Scrapped materials (part)

Inspection (where sampling inspection during manufacture or percentage
inspection applies)

Power for machines (major part)

On first analysis it would appear that these costs represent 7 to
17% of overheads in engineering and fabricating industries.

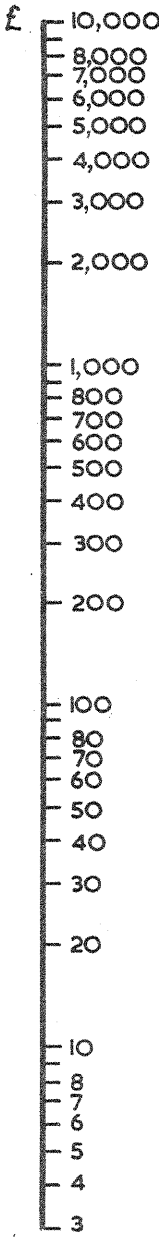
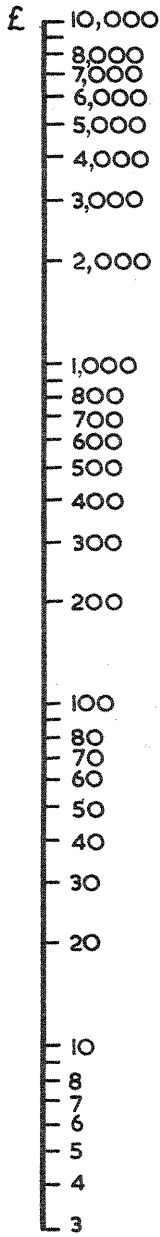
MEMOGRAM FOR CALCULATING Q

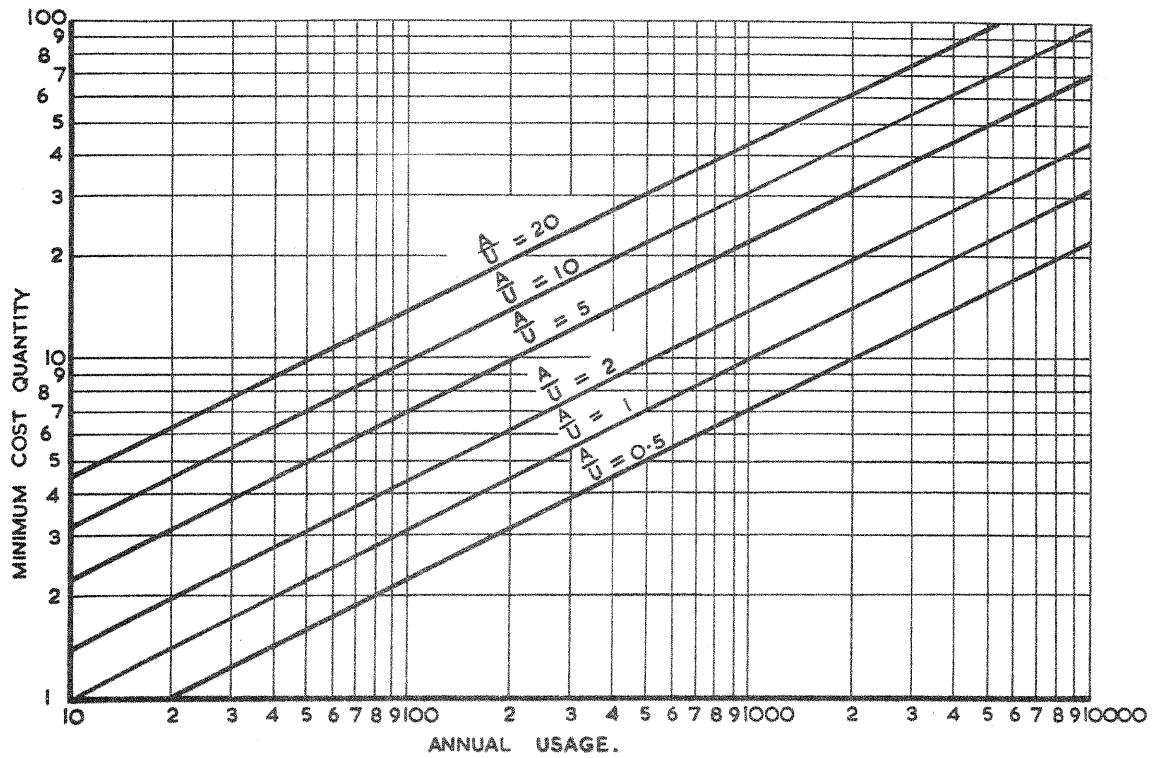
APP 4.

VALUE OF ANNUAL
USAGE (V_R)

VALUE OF ORDER
QUANTITY (Q_V)

m





MINIMUM COST QUANTITIES FOR GIVEN ANNUAL USAGES $I = 10^9$

